Scientific Research Publishing

Applied Mathematics, 2021, 12, 767-774
https://www.scirp.org/journal/am
ISSN Online: 2152-7393
ISSN Print: 2152-7385

Ideal Statistically Pre-Cauchy Triple Sequences of Fuzzy Number and Orlicz Functions

Xue Feng

College of Mathematics and Statistics, Qinghai Minzu University, Xining, China
Email: fengx8023@163.com

How to cite this paper: Feng, X. (2021) Ideal Statistically Pre-Cauchy Triple Sequences of Fuzzy Number and Orlicz Functions. *Applied Mathematics*, **12**, 767-774.
https://doi.org/10.4236/am.2021.129051

Received: July 28, 2021
Accepted: August 31, 2021
Published: September 3, 2021

Abstract

In this paper, we extend the notions of ideal statistically convergence for sequence of fuzzy number. We introduce the notions ideal statistically pre-Cauchy triple sequences of fuzzy number about Orlicz function, and give some correlation theorem. It is shown that $x=\{x_{ijk}\}$ is ideal statistically pre-Cauchy if and only if $\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\left|\left\{(i,j,k):D\left(x_{ijk},x_{pqr}\right)\geq\varepsilon,i\leq m,j\leq n,t\leq k\right\}\right|\geq\delta\right\}\in I$. At the same time, we have proved $x=\{x_{ijk}\}$ is ideal statistically convergent to x_0 if and only if $\left\{(m,n,t)\in N\times N\times N:\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D\left(x_{ijk},x_0\right)}{\rho}\right)\geq\delta\right\}\in I$. Also, some properties of these new sequence spaces are investigated.

Keywords

Fuzzy Numbers, Ideal Statistical Convergence, Orlicz Functions

1. Introduction

The notion of statistical convergence was introduced by Fast [1] and also independently by Buck [2] and Schoenberg [3] for real and complex sequences.Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory and Number theory. Later on it was further investigated from the sequence spaces point of view and linked with summability theory by Altinok and Et [4], Connor [5], Et *et al.* ([6] [7] [8]), Fridy [9], Fridy and Orhan [10], Mursaleen [11] and many others.

Matloka [12] defined the notion of fuzzy sequence and introduced bounded and convergent sequences of fuzzy real numbers and studied their some properties. After then, Nuray and Savas [13] defined the notion of statistical convergence for sequences of fuzzy numbers. Since then, there has been increasing interest in the study of statistical convergence of fuzzy sequences (see [14]-[19]).

Lindesstrauss and Tzafriri [20] used the idea of Orlicz sequence space, $l_M := \left\{ x \in \omega : \sum_{k=1}^{\infty} M\left(\frac{|x|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}$, which is Banachi space with the norm: $\|x\|_M = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x|}{\rho}\right) \le 1 \right\}$. The space l_M is closely related to the space l_p, which is an Orlicz sequence space with $M(x) = x^p$ for $1 \le p < \infty$.

Connor, Fridy and Kline [21] proved that statistical convergent sequences are statistically pre-Cauchy and any bounded statistically pre-Cauchy sequence with nowhere dense set of limit points is statistically convergent. They also gave an example showing statistically pre-Cauchy sequences are not necessarily statistically convergent.

In this paper, we extend the notions of ideal statistically convergence for sequence of fuzzy number. We introduce the notions ideal statistically pre-Cauchy triple sequences of fuzzy number about Orlicz function, and give some correlation theorem. Also, some properties of these new sequence spaces are investigated. It popularized the work of predecessors.

2. Definitions and Preliminaries

In this section, we give some basic notions which will be used throughout the paper.

Let $\tilde{A} \in \tilde{F}(R)$ be a fuzzy subset on *R*. If $\tilde{A}$ is convex, normal, upper semi-continuous and has compact support, we say that $\tilde{A}$ is a fuzzy number. Let $\tilde{R}^c$ denote the set of all fuzzy numbers.

For $\tilde{A} \in \tilde{R}^c$, we write the level set of $\tilde{A}$ as $A_\lambda = \{x : A(x) \ge \lambda\}$ and $A_\lambda = \left[A_\lambda^-, A_\lambda^+\right]$. Let $\tilde{A}, \tilde{B} \in \tilde{R}^c$, we define $\tilde{A} + \tilde{B} = \tilde{C}$ iff $A_\lambda + B_\lambda = C_\lambda$, $\lambda \in [0,1]$ iff $A_\lambda^- + B_\lambda^- = C_\lambda^-$ and $A_\lambda^+ + B_\lambda^+ = C_\lambda^+$ for any $\lambda \in [0,1]$. $A_\lambda \cdot B_\lambda = C_\lambda$, where

$$C_\lambda^- = \min\left\{ A_\lambda^- \cdot B_\lambda^-, A_\lambda^- \cdot B_\lambda^+, A_\lambda^+ \cdot B_\lambda^-, A_\lambda^+ \cdot B_\lambda^+ \right\},$$

$$C_\lambda^+ = \max\left\{ A_\lambda^- \cdot B_\lambda^-, A_\lambda^- \cdot B_\lambda^+, A_\lambda^+ \cdot B_\lambda^-, A_\lambda^+ \cdot B_\lambda^+ \right\}.$$

Define

$$D\left(\tilde{A}, \tilde{B}\right) = \sup_{\lambda \in [0,1]} d\left(A_\lambda, B_\lambda\right) = \sup_{\lambda \in [0,1]} \max\left\{ \left|A_\lambda^- - B_\lambda^-\right|, \left|A_\lambda^+ - B_\lambda^+\right| \right\},$$

where d is the Hausdorff metric. $D\left(\tilde{A}, \tilde{B}\right)$ is called the distance between $\tilde{A}$ and $\tilde{B}$.

Using the results of [22] [23], we see that

1) $\left(\tilde{R}^c, D\right)$ is a complete metric space,

2) $D(u+w, v+w) = D(u,v)$,

3) $D(ku,kv)=|k|D(u,v), k\in R$,

4) $D(u+v,w+e)\le D(u,w)+D(v,e)$,

5) $D(u+v,\bar{0})\le D(u,\bar{0})+D(v,\bar{0})$,

6) $D(u+v,w)\le D(u,w)+D(v+\bar{0})$,

Where $u,v,w,e\in \tilde{R}^c$, $\tilde{0}$ represents zero fuzzy number.

A sequence $\{x_n\}$ of fuzzy numbers is said to be statistically convergent to a fuzzy number x_0 if for each $\varepsilon>0$ the set $A(\varepsilon)=\{n\in N: D(x_n,x_0)\ge\varepsilon\}$ has natural density zero. The fuzzy number x_0 is called the statistical limit of the sequence $\{x_n\}$ and we write $st\text{-}\lim_{n\to\infty} x_n = x_0$ [24].

The concept of Orlicz function was introduced by Parashar and Choudhary [25], A mapping $M:[0,\infty)\to[0,\infty)$ is said to be an Orlicz funtion [26]

1) $M(0)=0$ iff $x=0$,

2) $M(x)>0$ for $x>0$,

3) $M(x)\to\infty$ as $x\to\infty$,

4) M is continuous, nondecreasing and convex.

An Orlicz function may be bounded or unbounded. For example,
$M(x)=x^p\ (0<p\le 1)$ is bounded.

A triple sequence can be defined as a function $X: N\times N\times N\to R(C)$ where N, R nad C denote the set of natural numbers, real numbers and complec numbers respectively. A triple sequence $\{x_{ijk}\}$ is said to be Cauchy sequence if for every $\varepsilon>0$, there exist $N(\varepsilon)\in N$ such that $|x_{ijk}-x_{pqr}|<\varepsilon$ whenever $i,p\ge N$, $j,q\ge N$, $k,r\ge N$ [27].

A triple sequence $\{x_{ijk}\}$ is called statistically pre-Cauchy if for every $\varepsilon>0$ there exist $p=p(\varepsilon),q(\varepsilon)$ and $r(\varepsilon)$ such that

$$\lim_{m,n,t\to\infty}\left|\frac{1}{m^2n^2t^2}\left|x_{ijk}-x_{pqr}\right|\ge\varepsilon, i\le m, j\le n, t\le k\right|=0.$$

where the vertical bars indicate the number of elements in the set [28].

3. Main Results

Definition 3.1. A triple sequence of fuzzy numbers is said to be ideal statistically pre-Cauchy if for every $\varepsilon>0,\delta>0$ there exist $p=p(\varepsilon),q(\varepsilon)$ and $r(\varepsilon)$ such that

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\left|\left\{(i,j,k):D(x_{ijk},x_{pqr})\ge\varepsilon, i\le m, j\le n, t\le k\right\}\right|\ge\delta\right\}\in I.$$

where the I denote the nontrivial ideal of N.

Theorem 3.1. Let $x=\{x_{ijk}\}$ be a triple sequence of fuzzy number and let M be a bounded Oricz function. Then x is ideal statistically pre-Cauchy if and only if

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le t}M\left(\frac{D(x_{ijk},x_{pqr})}{\rho}\right)\ge\delta\right\}\in I.$$

Proof. Suppose that

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le t}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)\ge\delta\right\}\in I.$$

For each $\varepsilon>0,\delta>0$ and $\rho>0$, $m,n,t\in N$, we have

$$\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le t}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$=\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le tD\left(x_{ijk},x_{pqr}\right)\le\varepsilon}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$+\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le tD\left(x_{ijk},x_{pqr}\right)\ge\varepsilon}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$\ge\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le tD\left(x_{ijk},x_{pqr}\right)\le\varepsilon}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$\ge M(\varepsilon)\left\{\frac{1}{m^2n^2t^2}\left|\left\{(i,j,k):D\left(x_{ijk},x_{pqr}\right)\ge\varepsilon,i\le m,j\le n,k\le t\right\}\right|\ge\delta\right\}\in I.$$

Now suppose that x is ideal statistically pre-Cauchy and that ε has been given.

Let $\varepsilon>0,\delta>0$ be such that $M(\xi)<\frac{\varepsilon}{2}$.

Since M is bounded Orlicz function, there exist an integer G such that $M(x)<\frac{G}{2}$ for all $x\ge 0$.

Not that, for each $m,n,t\in N$

$$\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le t}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$=\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le tD\left(x_{ijk},x_{pqr}\right)<\xi}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$+\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le tD\left(x_{ijk},x_{pqr}\right)\ge\xi}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$\le M(\xi)+\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le tD\left(x_{ijk},x_{pqr}\right)\ge\xi}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)$$

$$\le\frac{\varepsilon}{2}+\frac{G}{2}\left\{\frac{1}{m^2n^2t^2}\left|\left\{(i,j,k):D\left(x_{ijk},x_{pqr}\right)\ge\xi,i\le m,j\le n,k\le t\right\}\right|\ge\delta\right\}$$

$$\le\varepsilon+G\left\{\frac{1}{m^2n^2t^2}\left|\left\{(i,j,k):D\left(x_{ijk},x_{pqr}\right)\ge\xi,i\le m,j\le n,k\le t\right\}\right|\ge\delta\right\}\in I.$$

Hence

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i,p\le m}\sum_{j,q\le n}\sum_{k,r\le t}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)\ge\delta\right\}\in I.$$

Theorem 3.2. Let $x=\left\{x_{ijk}\right\}$ be a triple sequence of fuzzy numbers and let M be a bounded Orlicz function. Then x is ideal statistically convergent to x_0 if and only if

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\geq\delta\right\}\in I.$$

Proof. Suppose that

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\geq\delta\right\}\in I.$$

For each $\varepsilon>0,\delta>0$ and $\rho>0$, $m,n,t\in N$, we have

$$\begin{aligned}
&\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&=\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1,D(x_{ijk},x_0)\geq\varepsilon}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&\quad+\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1,D(x_{ijk},x_0)<\varepsilon}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&\geq\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1,D(x_{ijk},x_0)\geq\varepsilon}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&\geq M(\varepsilon)\left\{\frac{1}{mnt}\left|\left\{(i,j,k):D(x_{ijk},x_0)\geq\varepsilon,i\leq m,j\leq n,k\leq t\right\}\right|\geq\delta\right\}\in I.
\end{aligned}$$

We have x is ideal statistically convergent to x_0.

Now suppose that x is ideal statistically convergent to x_0, let $\varepsilon>0,\delta>0$ be such that $M(\xi)<\frac{\varepsilon}{2}$.

Since M is bounded Orlicz function, there exist an integer G such that $M(X)<\frac{G}{2}$ for all $x\geq 0$.

Note that, for each $m,n,t\in N$

$$\begin{aligned}
&\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&=\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1,D(x_{ijk},x_0)\geq\varepsilon}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&\quad+\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1,D(x_{ijk},x_0)<\varepsilon}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&\leq M(\xi)+\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1,D(x_{ijk},x_0)\geq\varepsilon}^{t}M\left(\frac{D(x_{ijk},x_0)}{\rho}\right)\\
&\leq\frac{\varepsilon}{2}+\frac{G}{2}\left\{\frac{1}{mnt}\left|\left\{(i,j,k):D(x_{ijk},x_{pqr})\geq\xi,i\leq m,j\leq n,k\leq t\right\}\right|\geq\delta\right\}\\
&\leq\varepsilon+G\left\{\frac{1}{mnt}\left|\left\{(i,j,k):D(x_{ijk},x_{pqr})\geq\xi,i\leq m,j\leq n,k\leq t\right\}\right|\geq\delta\right\}\in I.
\end{aligned}$$

Hence

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D\left(x_{ijk},x_0\right)}{\rho}\right)\geq\delta\right\}\in I.$$

Corollary 3.3. Let $x=\{x_{ijk}\}$ be a bound triple sequence of fuzzy number. Then x is ideal statistically pre-Cauchy if and only if

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}D\left(x_{ijk},x_{pqr}\right)\geq\delta\right\}\in I.$$

Proof. Let $K=\sup_{i,j,k}D\left(x_{ijk},\overline{0}\right)$ and defined $M(x)=\frac{(1+2K)x}{1+x}$, then $M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)\leq(1+2K)D\left(x_{ijk},x_{pqr}\right)$, and

$$\begin{aligned}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)&=(1+2K)\frac{D\left(x_{ijk},x_{pqr}\right)}{1+D\left(x_{ijk},x_{pqr}\right)}\\&\geq\frac{\frac{(1+2K)D\left(x_{ijk},x_{pqr}\right)}{\rho}}{1+D\left(x_{ijk},x_{pqr}\right)}\\&\geq\frac{(1+2K)D\left(x_{ijk},x_{pqr}\right)}{1+2A}=D\left(x_{ijk},x_{pqr}\right).\end{aligned}$$

Hence $\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}D\left(x_{ijk},x_{pqr}\right)\geq\delta\right\}\in I$, if and only if $\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i,p\leq m}\sum_{j,k\leq n}\sum_{t,r\leq t}M\left(\frac{D\left(x_{ijk},x_{pqr}\right)}{\rho}\right)\geq\delta\right\}\in I$, and an immediate application of Theorem 3.1 completes the proof.

Corollary 3.4. Let $x=\{x_{ijk}\}$ be a bound triple sequence of fuzzy number. Then x is ideal statistically convergent x_0 if and only if

$$\left\{(m,n,t)\in N\times N\times N:\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}D\left(x_{ijk},x_0\right)\geq\delta\right\}\in I.$$

Proof. Let $K=\sup_{i,j,k}D\left(x_{ijk},\overline{0}\right)$ and defined $M(x)=\frac{(1+K+x_0)x}{1+x}$, then $M\left(\frac{D\left(x_{ijk},x_0\right)}{\rho}\right)\leq(1+K+x_0)D\left(x_{ijk},x_0\right)$, and

$$\begin{aligned}M\left(\frac{D\left(x_{ijk},x_0\right)}{\rho}\right)&=(1+K+x_0)\frac{D\left(x_{ijk},x_0\right)}{1+D\left(x_{ijk},x_0\right)}\\&\geq\frac{\frac{(1+K+x_0)D\left(x_{ijk},x_0\right)}{\rho}}{1+D\left(x_{ijk},x_0\right)}\\&\geq\frac{(1+K+x_0)D\left(x_{ijk},x_0\right)}{1+K+x_0}=D\left(x_{ijk},x_0\right).\end{aligned}$$

Hence $\left\{(m,n,t)\in N\times N\times N:\frac{1}{mnt}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}D\left(x_{ijk},x_0\right)\geq\delta\right\}\in I$, if and only if $\left\{(m,n,t)\in N\times N\times N:\frac{1}{m^2n^2t^2}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{t}M\left(\frac{D\left(x_{ijk},x_0\right)}{\rho}\right)\geq\delta\right\}\in I$, and an immediate application of Theorem 3.1 completes the proof.

4. Conclusion

In this article, we introduced ideal statistically pre-Cauchy triple sequences of fuzzy numbers about Orlicz function. At the same time, we have proved some properties and relationships.

Fund

This work is supported by National Natural Science Fund of China (11761056); the Natural Science Foundation of Qinghai Province (2020-ZJ-920); University level planning project of Qinghai Minzu University (2021XJGH24).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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